Origin of Structure in a Supersymmetric Quantum Universe

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In this report we advance the current repertoire of quantum cosmological models to incorporate inhomogenous field modes in a supersymmetric manner. In particular, we introduce perturbations about a supersymmetric FRW model. A quantum state of our model has properties typical of the *no-boundary* (Hartle–Hawking) proposal. This solution may then lead to a scale–free spectrum of density perturbations.

The main objective of this report is to examine *if* and *how* the inclusion of supersymmetry in a quantum cosmological scenario may add to our understanding of structure formation in the very early universe. Supersymmetry is a transformation which relates bosons and fermions and induces the cancelation of divergences that are otherwise present in plain quantum gravity. Thus, its presence seems to constitute an element of remarkable value.

The fundamental element of our approach is that N=1 supergravity constitutes a "square-root" [1] of gravity: in finding a physical state Ψ, it is sufficient to solve the Lorentz and supersymmetry constraints; Ψ will consequently obey the Hamiltonian constraints.[a] In more precise terms, simple *first-order* differential equations have to be solved. This contrasts with the situation without supersymmetry: *second-order* Wheeler-DeWitt equation has to be solved, employing boundary conditions [2].

The action for our model is retrieved from the general action of the theory of N=1 of supergravity[3] with scalar supermultyplets. Our *background* supersymmetric FRW minisuperspace is described by a tetrad $e_\mu^{AA'} = e_\mu^a \sigma_a^{AA'}$ with $e_{a\mu} = \mathrm{diag}[N(t), a(t)E_{\hat{a}i}]$, where $\hat{a}$ and i run from 1 to 3, $E_{\hat{a}i}$ is a basis of left-invariant 1-forms on the unit S^3 and $N(t)$, $a(t)$, $\sigma_a^{AA'} (A = 0, 1)$ denote respectively the lapse function, scale factor and Infeld–Van der Warden symbols. The gravitinos must have the form[6] $\psi^A{}_i = e^{AA'}{}_i \bar{\psi}_{A'}(t)$, $\bar{\psi}^{A'}{}_i = e^{AA'}{}_i \psi_A(t)$, where $\psi_A, \bar{\psi}_{A'}$ constitute time-dependent spinor fields and $\psi_0^A(t), \bar{\psi}_0^{A'}(t)$ are Lagrange multipliers. The "overline" denotes Hermitian conjugation. A set of time-dependent complex scalar fields, $\phi, \bar{\phi}$, and their fermionic superpartners, $\chi_A(t), \bar{\chi}_{A'}(t)$ are also included.

As far as the perturbations about the background minisuperspace are concerned, we take the scalar fields as

$$\Phi(x_i, t) = \phi(t) + \Sigma_{nlm} f_n^{lm}(t) Q_{lm}^n(x_i), \tag{1}$$

with its Hermitian conjugate where the coefficients f_n^{lm} are functions of the time coordinate t and Q_{lm}^n are standard scalar spherical harmonics[4] on S^3. The fermionic

[a] For a review on canonical quantization of supersymmetric minisuperspaces see ref. [6].

superpartners are expanded as (see ref. [5]):

$$\mathbf{X}_A(t,x_i) = \chi_A(t) + a^{-3/2}\Sigma_{mpq}\beta_m^{pq}\left[s_{mp}(t)\rho_A^{nq}(x_i) + \bar{t}_{mp}(t)\bar{\tau}_A^{mq}(x_i)\right], \tag{2}$$

with $\rho_A^{mq}, \bar{\tau}_A^{mq}$ are spinor hyperspherical harmonics on S^3.

We obtain simple canonical relations (i.e., Dirac brackets)

$$[\chi_A, \bar{\chi}_B]_D = -i\epsilon_{AB}\ ,\ [\psi_A, \bar{\psi}_B]_D = i\epsilon_{AB}, [a, \pi_a]_D = 1\ ,\ [\phi, \pi_\phi]_D = 1\ ,\ [\bar{\phi}, \pi_{\bar{\phi}}]_D = 1, \tag{3}$$

where ϵ_{AB} is the alternating spinor [6]. All other bracket relations yield zero with the exception of

$$[f_n, \pi_{f_n}]_D = \delta_{mn},\ [\bar{f}_n, \pi_{\bar{f}_n}]_D = \delta_{mn},\ [s_{np}, \bar{s}_{n'p'}]_D = -i\delta_{nn'}\delta_{pp'},\ [t_{np}, \bar{t}_{n'p'}]_D = -i\delta_{nn'}\delta_{pp'}. \tag{4}$$

En route to a quantum mechanical description of our model, the coordenates of our minisuperspace are chosen to be $\left(\chi_A, \psi_A, a, \phi, \bar{\phi}; f_n^{lm}, \bar{f}_n^{lm}, s_{np}, t_{np}\right)$ while $\left(\bar{\chi}_A, \bar{\psi}_A, \pi_a, \pi_\phi, \pi_{\bar{\phi}}; \pi_{f_n^{lm}}, \pi_{\bar{f}_n^{lm}}, \bar{s}_{np}, \bar{t}_{np}\right)$ constitute the canonical momentum variables.

A natural ansatz for the wave function of the universe has the form

$$\begin{aligned}
\Psi &= A + B\psi^C\psi_C + iC\psi^C\chi_C + D\chi^D\chi_D + E\psi^C\psi_C\chi^D\chi_D \\
&= A^{(0)}(a,\phi,\bar{\phi})\Pi_n A^{(n)}(a,\bar{\phi},\phi; f_n\bar{f}_n)\Pi_m A^{(m)}(a,\phi,\bar{\phi},s_m,t_m) \\
&+ B^{(0)}(a,\phi,\bar{\phi})\Pi_n B^{(n)}(a,\bar{\phi},\phi; f_n\bar{f}_n)\Pi_m B^{(m)}(a,\phi,\bar{\phi},s_m,t_m)\psi^C\psi_C \\
&+ C^{(0)}(a,\phi,\bar{\phi})\Pi_n C^{(n)}(a,\bar{\phi},\phi; f_n\bar{f}_n)\Pi_m C^{(m)}(a,\phi,\bar{\phi},s_m,t_m)\psi^C\chi_C \\
&+ D^{(0)}(a,\phi,\bar{\phi})\Pi_n D^{(n)}(a,\bar{\phi},\phi; f_n\bar{f}_n)\Pi_m D^{(m)}(a,\phi,\bar{\phi},s_m,t_m)\chi^C\chi_C \\
&+ E^{(0)}(a,\phi,\bar{\phi})\Pi_n E^{(n)}(a,\bar{\phi},\phi; f_n\bar{f}_n)\Pi_m E^{(m)}(a,\phi,\bar{\phi},s_m,t_m)\psi^C\psi_C\chi^D\chi_D,
\end{aligned} \tag{5}$$

where each bosonic coefficient $A^{(n)}, A^{(m)}$, ..., $E^{(n)}, E^{(m)}$ depends either on the individual perturbation modes f_n or s_m, t_m. The expression (5) satisfies the Lorentz constraints associated with the unperturbed field variables $\psi_A, \bar{\psi}_A, \chi_A$ and $\bar{\chi}_A$: $J_{AB} = \psi_{(A}\bar{\psi}_{B)} - \chi_{(A}\bar{\chi}_{B)} = 0$. The coefficients $s_m, t_m, \bar{s}_m, \bar{t}_m$, will be taken as invariant under local Lorentz transformation to lowest order in perturbation. Overall, this approach is fully satisfactory and indeed we can extract consistent set of solutions. From the supersymmetry constraints $S_A\Psi = 0$ and $\bar{S}_A\Psi = 0$, we then obtain (see ref. [9] for more details) the following solutions:

$$\begin{aligned}
E^{(0)} &= \hat{E}_0^{(0)}\frac{e^{3a^2+\phi(2\lambda_6-\Omega_5)-\Omega_5\bar{\phi}}}{a^{\Omega_6}} && (6)\\
E^{(n)} &= E_0^{(n)}e^{-\lambda_7\bar{\phi}+\phi(2\lambda_8-\lambda_7)}e^{2\lambda_9\bar{f}_n+2a^2(n-1)f_n\bar{f}_n-(\Omega_7-\lambda_9)f_n+(\Omega_7-\lambda_9)\bar{f}_n}, && (7)\\
E^{(m)} &= E_0^{(m)}e^{2\lambda_8\bar{\phi}-C_2\phi\bar{\phi}-\Omega_9\phi+\Omega_9\bar{\phi}}\tilde{E}, && (8)
\end{aligned}$$

where $\hat{E}_0^{(0)} = E_0^{(0)}e^{-3a^2}$ $E_0^{(n)}, E_0^{(m)}$ denote integration constants and $\tilde{E} \sim s_{mp}$ or t_{mp}. The quantities Ω_1, Ω_2, ... represent back reactions of the scalar and fermionic perturbed modes in the homogenous modes and are assumed to be of a very small value.

Charactheristic features of the no-boundary (Hartle-Hawking) solution are present in the bosonic coefficient E (6)-(8) (see ref. [4,5,6]). This state requires $|\Omega_6| \ll 1$

and the term $e^{-na^2 f_n \bar{f}_n}, (n \gg 1)$ in eq. (7) to dominate over the other remaining exponential terms. It seems thus that supersymmetry selects the no-boundary (Hartle-Hawking) quantum state as mandatory. Since such wave function may lead to a satisfactory spectrum of density perturbations [4], it thus seems that supersymmetry in the very early universe intrisically contains the relevant seeds for structure formation.

But do the results hereby presented contribute to our understanding of the very early universe and if yes, how? Within our context, the answer to those questions is then a *yes* but where some caution is nevertheless required. In conclusion, we have shown that by employing supersymmetry in a perturbed quantum cosmological model we can extract significant information about the very early universe evolution. Supersymmetric quantum cosmology is a forefront line of research and this essay's results point to further analysis which surely will have a considerable impact on our understanding of the very early universe.

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